Long-Distance Resolution: Proof Generation and Strategy Extraction in Search-Based QBF Solving

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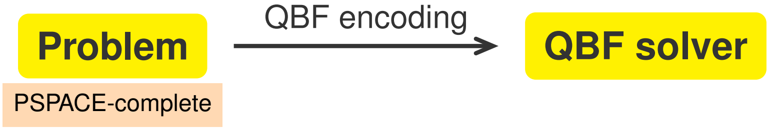
Overview



Overview

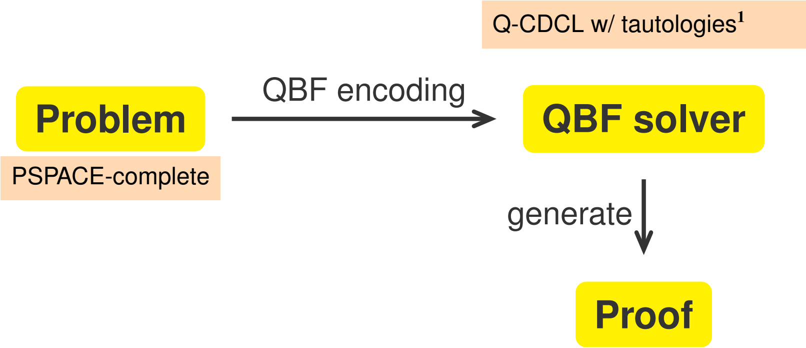
Q-CDCL w/ tautologies[1]

Problem —QBF encoding→ **QBF solver**

PSPACE-complete

[1] Zhang and Malik, 2002

Overview



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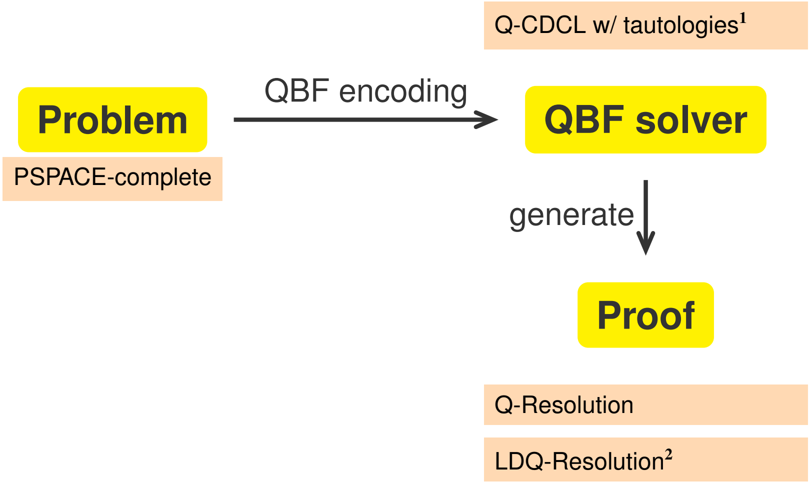

[1] Zhang and Malik, 2002
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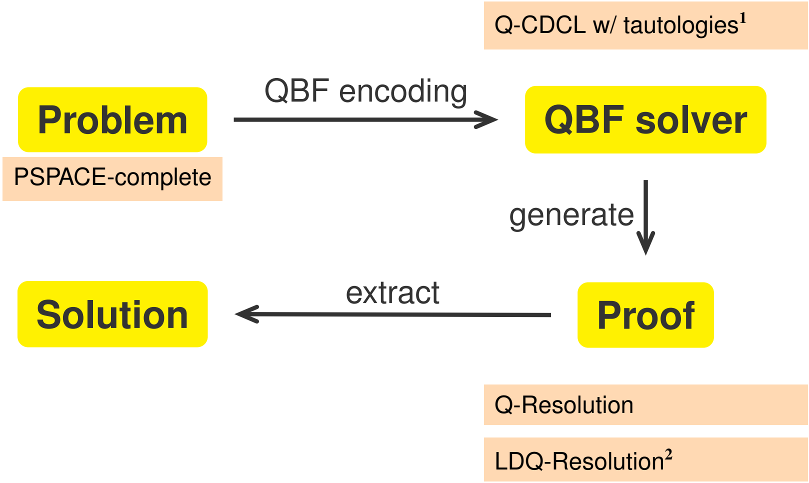

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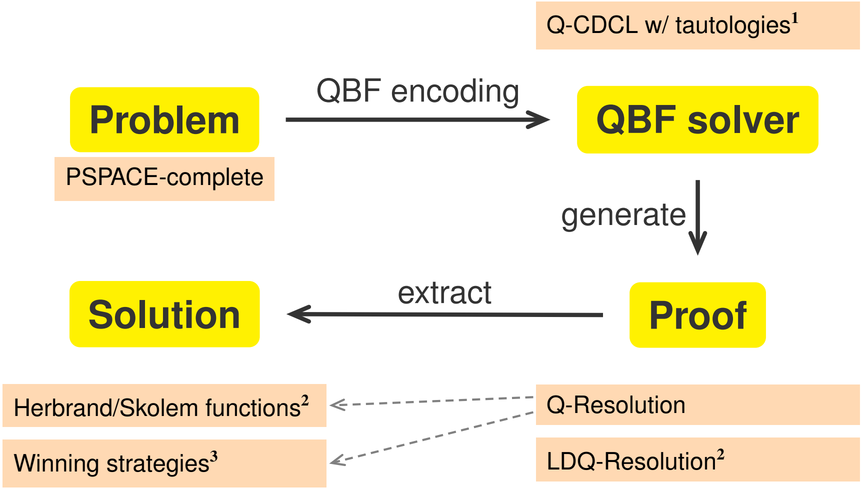

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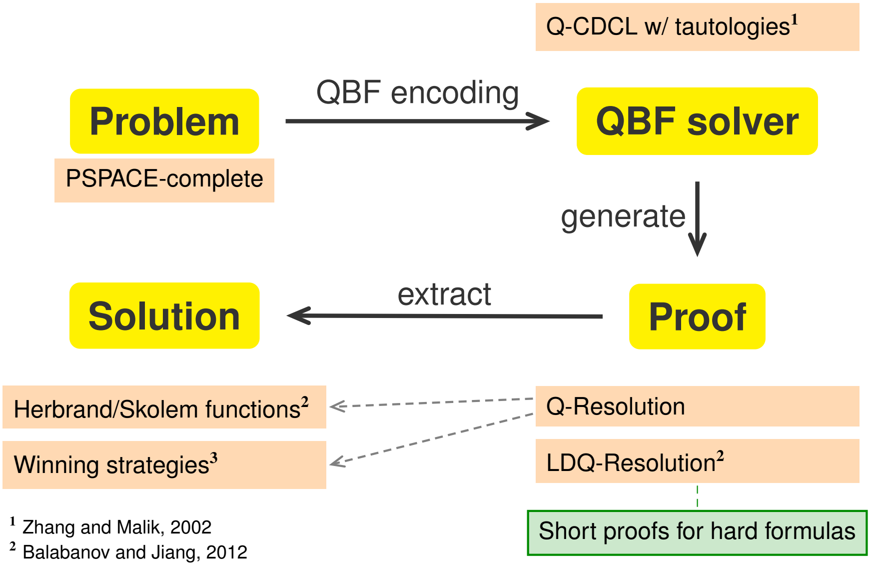

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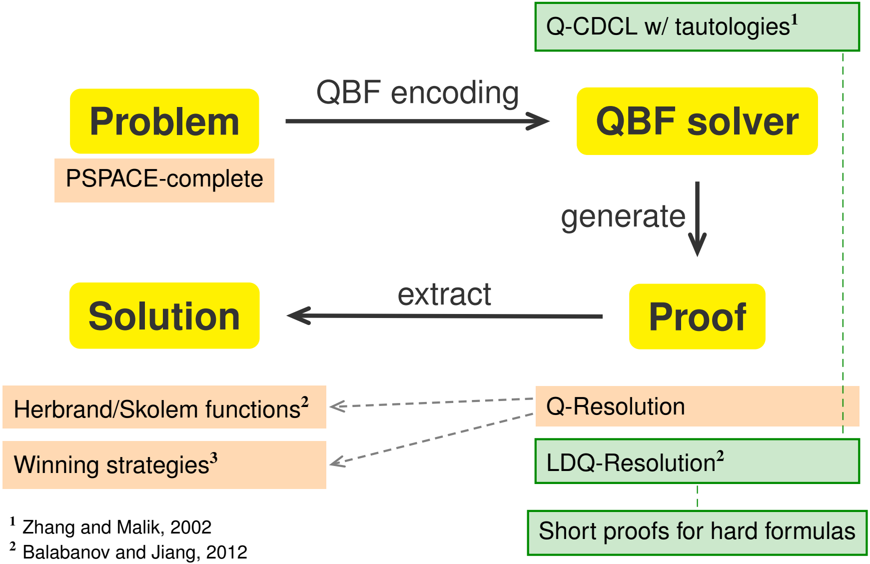

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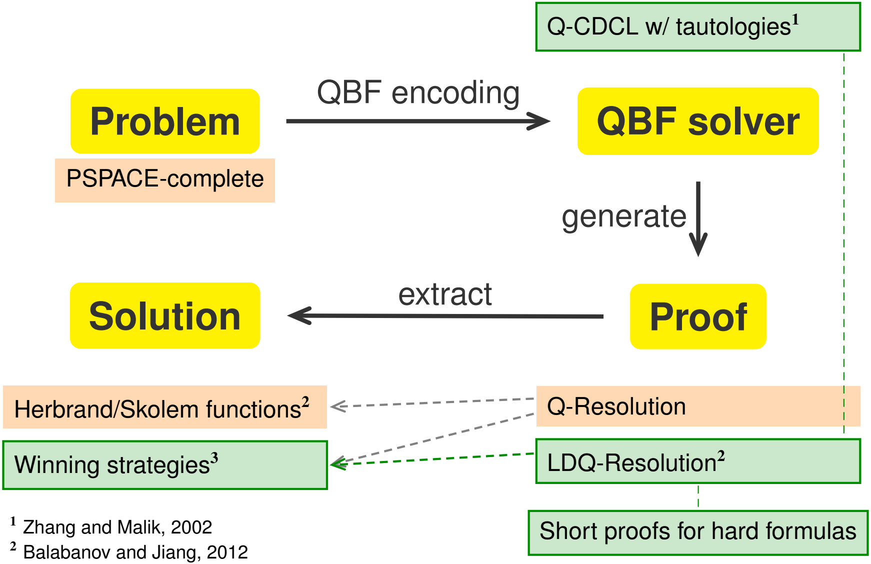

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Quantified Boolean Formulas (QBF)

$$\psi := Q_1 x_1 \dots Q_n x_n.\ \phi$$

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$Q_i \in \{\exists, \forall\}$ Prop. formula in CNF

$$\forall x \exists y .\ (x \vee \neg y) \wedge (\neg x \vee y)$$

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Recursive QBF Semantics

- Assign variables in prefix order (from left to right)
- Base cases: the QBF $\top$ ($\bot$) is true (false).
- $\psi = \forall x \ldots \phi$ is true if $\psi[x/\bot]$ and $\psi[x/\top]$ are true.
- $\psi = \exists x \ldots \phi$ is true if $\psi[x/\bot]$ or $\psi[x/\top]$ is true.

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$$\neq$$

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Long-distance (LDQ) resolution

$\exists a, b, c\ \forall x\ \exists d, e$

$(d) \quad (\bar{a}, e) \quad (a, e) \quad (b, x, \bar{d}, \bar{e}) \quad (c, \bar{x}, \bar{d}, \bar{e}) \quad (\bar{b}, \bar{c})$

Long-distance (LDQ) resolution

$$\exists a, b, c\ \forall x\ \exists d, e$$

$$(d) \qquad (\bar{a}, e) \qquad (a, e) \qquad (b, x, \bar{d}, \bar{e}) \qquad (c, \bar{x}, \bar{d}, \bar{e}) \qquad (\bar{b}, \bar{c})$$

$$(a, b, x, \bar{d})$$

Long-distance (LDQ) resolution

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Long-distance (LDQ) resolution

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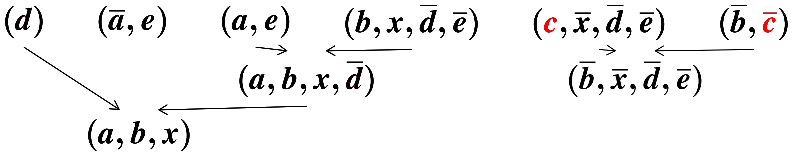


Long-distance (LDQ) resolution

$\exists a, b, c \ \forall x \ \exists d, e$

$(d) \quad (\bar{a}, e) \quad (a, e) \quad (b, x, \bar{d}, \bar{e}) \quad (c, \bar{x}, \bar{d}, \bar{e}) \quad (\bar{b}, \bar{c})$

$(a, e), (b, x, \bar{d}, \bar{e}) \rightarrow (a, b, x, \bar{d})$

$(c, \bar{x}, \bar{d}, \bar{e}), (\bar{b}, \bar{c}) \rightarrow (\bar{b}, \bar{x}, \bar{d}, \bar{e})$

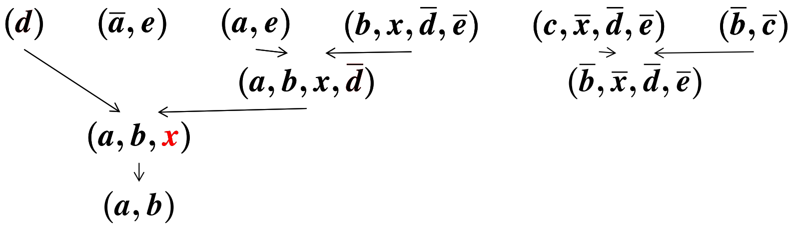
$(d), (a, b, x, \bar{d}) \rightarrow (a, b, \mathbf{x})$

$(a, b, x) \downarrow (a, b)$

Long-distance (LDQ) resolution

All ∃ variables in the clause are quantified left of reduced variable

$$\exists a, b, c\ \forall x\ \exists d, e$$



Long-distance (LDQ) resolution

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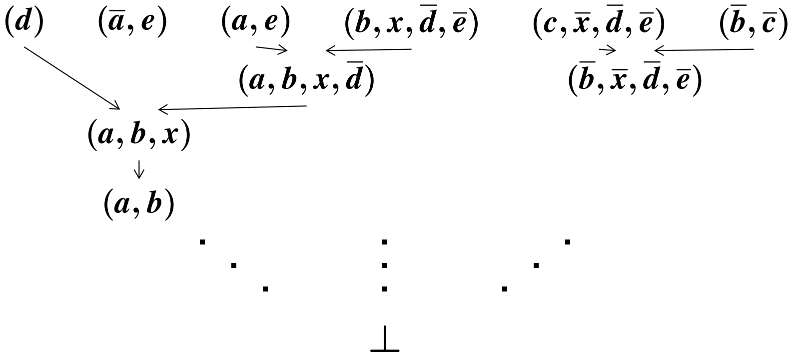
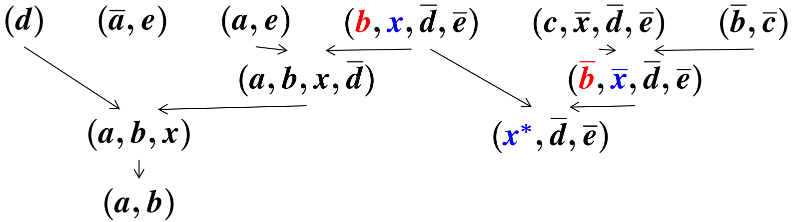


Long-distance (LDQ) resolution

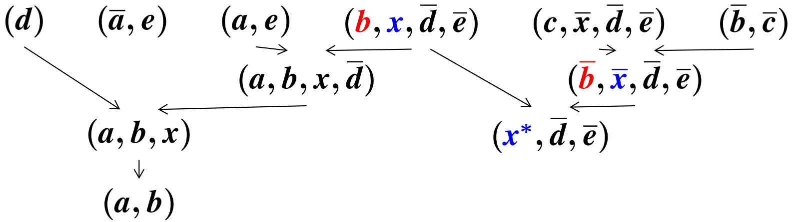
$\exists a, b, c\ \forall x\ \exists d, e$



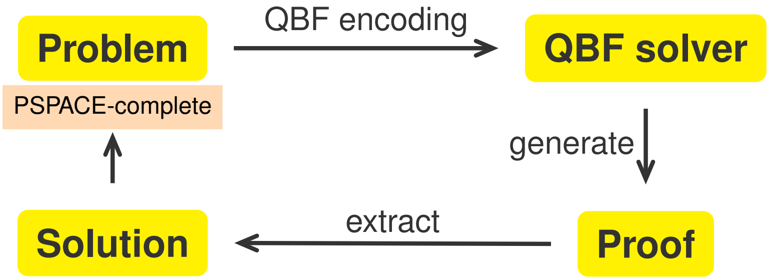
Long-distance (LDQ) resolution

Pivot variable left of merged universal variable

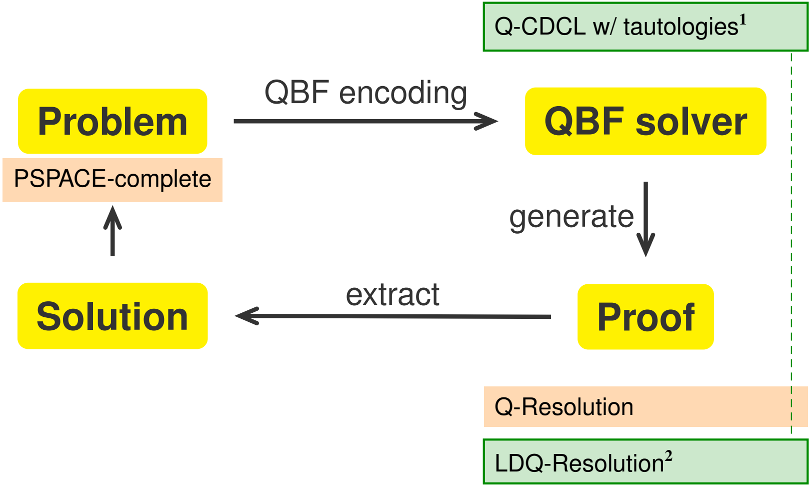
$$\exists a, b, c \; \forall x \; \exists d, e$$



Overview


Problem
QBF encoding
QBF solver
PSPACE-complete
generate
Solution
extract
Proof

Overview



[1] Zhang and Malik, 2002
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Proof Generation

$$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\bar{a}, e) \quad (a, e) \quad (b, x, \bar{d}, \bar{e}) \quad (c, \bar{x}, \bar{d}, \bar{e}) \quad (\bar{b}, \bar{c})$$

$$\sigma := \{\}$$

Proof Generation

$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\bar{a}, e) \quad (a, e) \quad (b, x, \bar{d}, \bar{e}) \quad (c, \bar{x}, \bar{d}, \bar{e}) \quad (\bar{b}, \bar{c})$

$\sigma = \{d\}$ (unit assignment)

$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\bar{a}, e) \quad (a, e) \quad (b, x, \bar{d}, \bar{e}) \quad (c, \bar{x}, \bar{d}, \bar{e}) \quad (\bar{b}, \bar{c})$

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$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\overline{a}, e) \quad (a, e) \quad (b, x, \overline{d}, \overline{e}) \quad (c, \overline{x}, \overline{d}, \overline{e}) \quad (\overline{b}, \overline{c})$

$\sigma := \{d, \overline{a}\}$ (tentative assignment)

$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\overline{a}, e) \quad (a, e) \quad (b, x, \overline{d}, \overline{e}) \quad (c, \overline{x}, \overline{d}, \overline{e}) \quad (\overline{b}, \overline{c})$

Proof Generation

$\exists a,b,c \forall x \exists d,e \quad (d) \quad (\bar{a},e) \quad (a,e) \quad (b,x,\bar{d},\bar{e}) \quad (c,\bar{x},\bar{d},\bar{e}) \quad (\bar{b},\bar{c})$

$\sigma := \{d,\bar{a},e\}$ (unit assignment)

$\exists a,b,c \forall x \exists d,e \quad (d) \quad (\bar{a},e) \quad (a,e) \quad (b,x,\bar{d},\bar{e}) \quad (c,\bar{x},\bar{d},\bar{e}) \quad (\bar{b},\bar{c})$

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$\sigma := \{d, \bar{a}, e\}$ (universal reduction)

$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\bar{a}, e) \quad (a, e) \quad (b, x, \bar{d}, \bar{e}) \quad (c, \bar{x}, \bar{d}, \bar{e}) \quad (\bar{b}, \bar{c})$

Proof Generation

$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\bar{a}, e) \quad (a, e) \quad (b, x, \bar{d}, \bar{e}) \quad (c, \bar{x}, \bar{d}, \bar{e}) \quad (\bar{b}, \bar{c})$

$\sigma := \{d, \bar{a}, e, b, c\}$ (unit assignment)

$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\bar{a}, e) \quad (a, e) \quad (b, x, \bar{d}, \bar{e}) \quad (c, \bar{x}, \bar{d}, \bar{e}) \quad \bot$

Proof Generation

$\exists a, b, c \forall x \exists d, e \quad (d) \quad (\overline{a}, e) \quad (a, e) \quad (b, x, \overline{d}, \overline{e}) \quad (c, \overline{x}, \overline{d}, \overline{e}) \quad (\overline{b}, \overline{c})$

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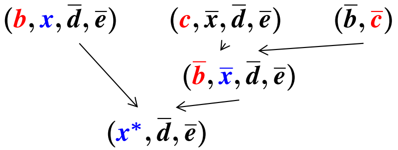

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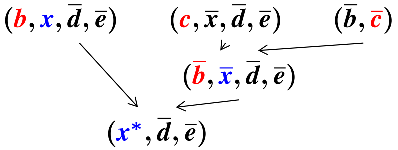
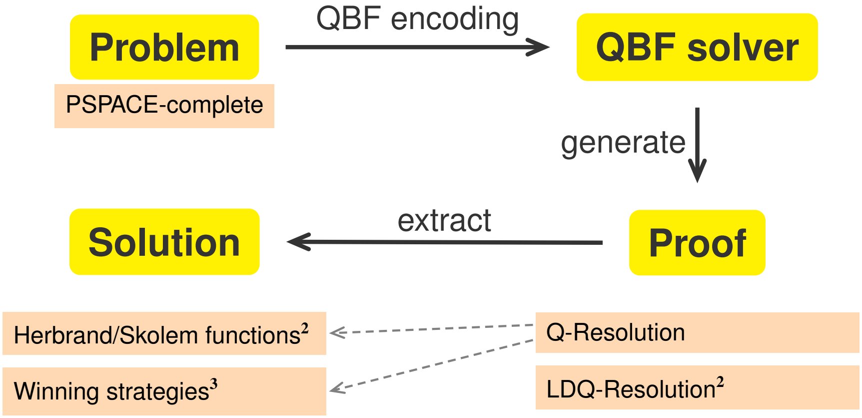


- Learning of tautological clauses was first applied in QBF solver “yquaffle”.
- This procedure follows the LDQ-calculus.
- Promising experimental results for this procedure in DepQBF.

Overview



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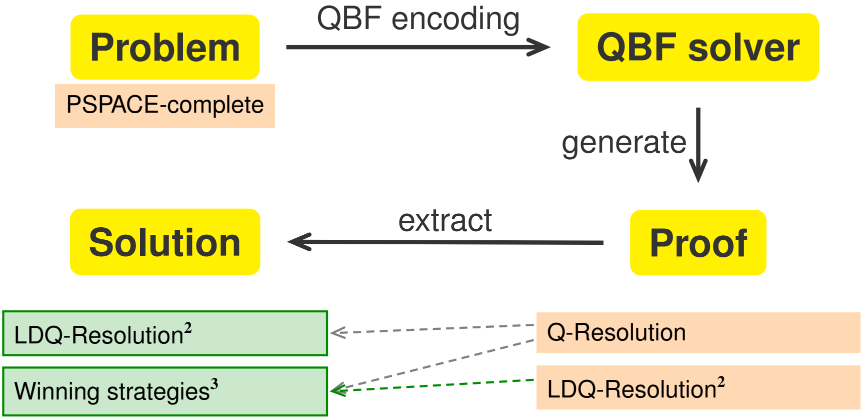

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Strategy Extraction from LDQ-Refutations

$\exists a \ \forall x \ \exists b \ \forall y \ \exists c$

(a, x, b, y, c) $\quad$ $(\bar{a}, \bar{y}, c)$ $\quad$ $(\bar{c})$ $\quad$ $(\bar{b})$

Strategy Extraction from LDQ-Refutations

$\exists a \ \forall x \ \exists b \ \forall y \ \exists c$

LDQ-Resolution

$(a, x, b, y, c) \qquad (\overline{a}, \overline{y}, c) \qquad (\overline{c}) \qquad (\overline{b})$

$(a, x, b, y, c),\ (\overline{a}, \overline{y}, c) \rightarrow (x, b, y^*, c)$

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LDQ-Resolution

(a, x, b, y, c) $(\bar{a}, \bar{y}, c)$ $(\bar{c})$ $(\bar{b})$

(x, b, y^*, c)

(x, b, y^*)

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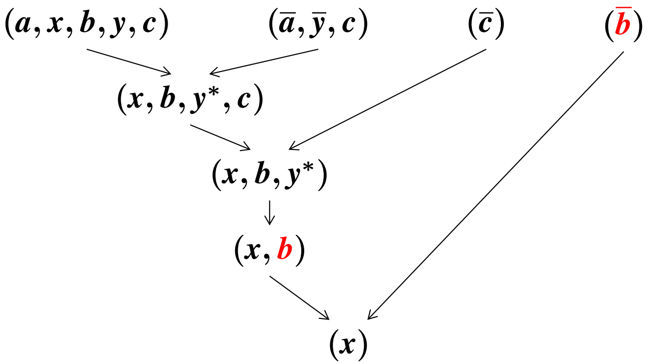

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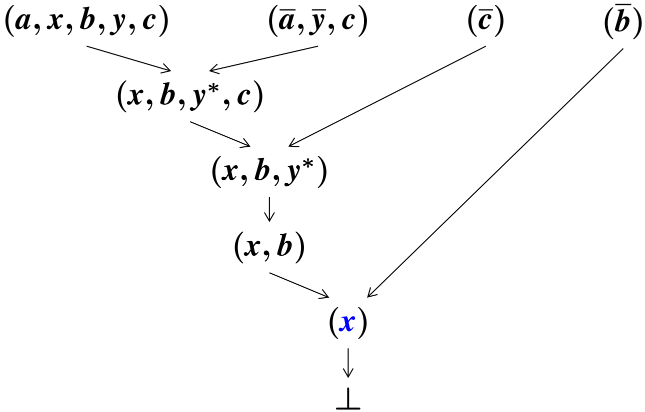

Strategy Extraction from LDQ-Refutations

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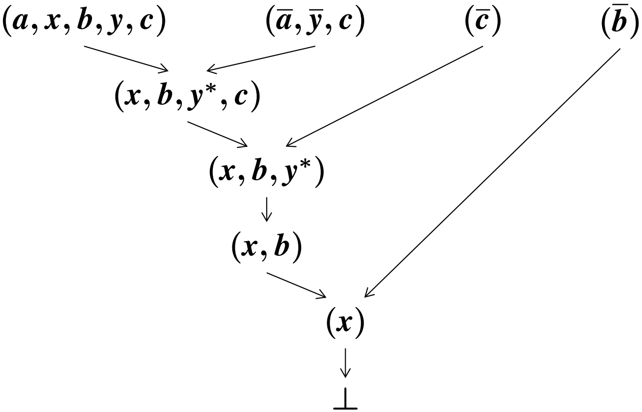

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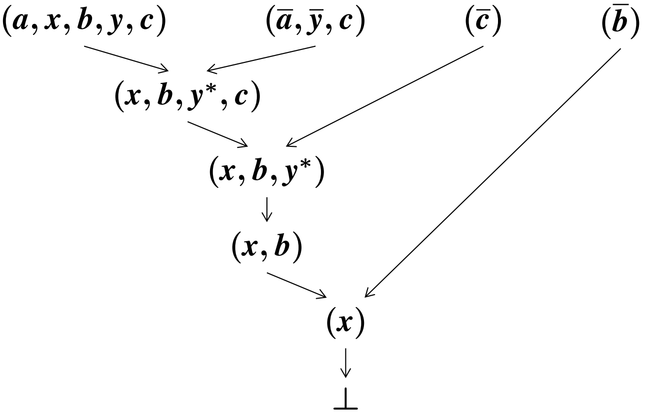
Strategy Extraction



Strategy Extraction from LDQ-Refutations

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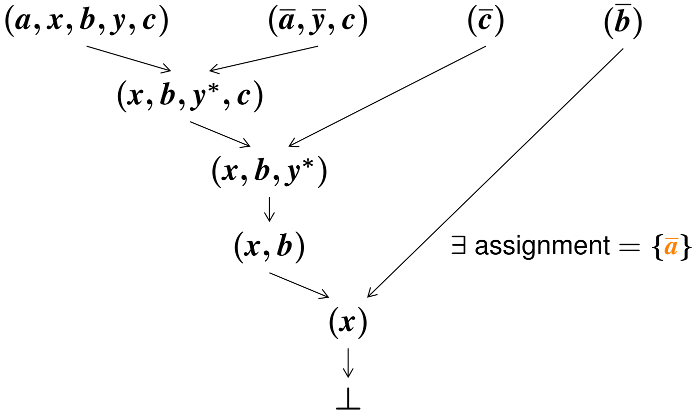
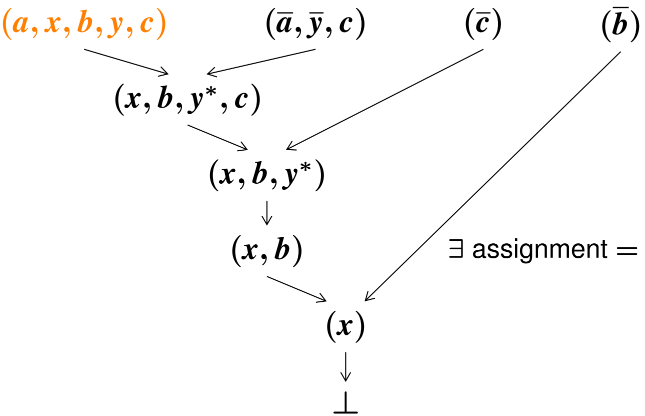
Strategy Extraction



Strategy Extraction from LDQ-Refutations

$\exists a\ \forall x\ \exists b\ \forall y\ \exists c$

Strategy Extraction

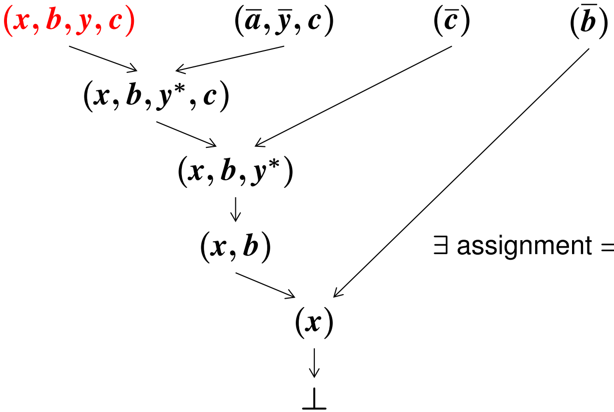


$\exists$ assignment $= \{\bar{a}\}$

Strategy Extraction from LDQ-Refutations

Strategy Extraction

$\exists a \; \forall x \; \exists b \; \forall y \; \exists c$



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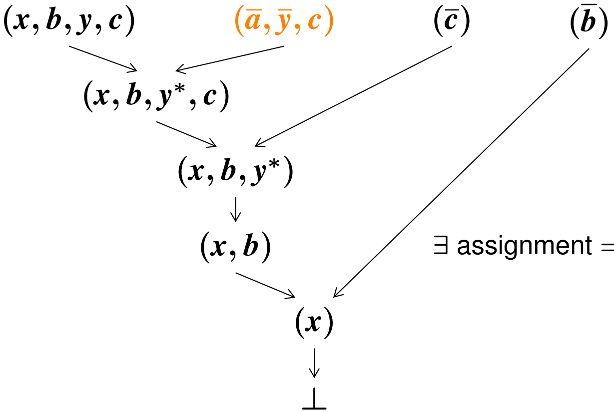

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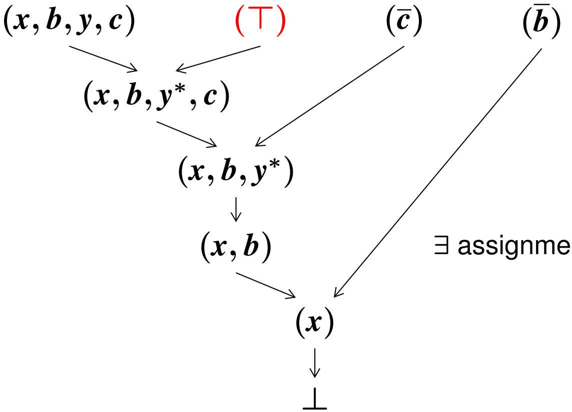

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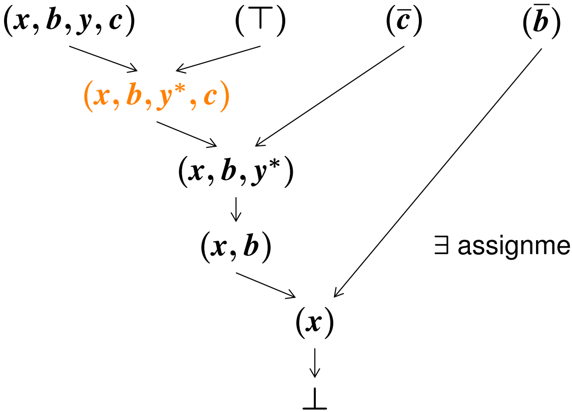

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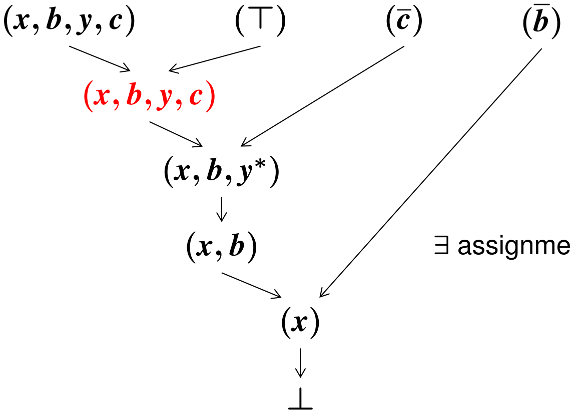


$\exists$ assignment $= \{\bar{a}\}$

Strategy Extraction from LDQ-Refutations

$\exists a \; \forall x \; \exists b \; \forall y \; \exists c$

Strategy Extraction

$(x, b, y, c) \quad (\top) \quad (\bar{c}) \quad (\bar{b})$

(x, b, y, c)

(x, b, y^*)

(x, b)

$\exists$ assignment $= \{\bar{a}\}$

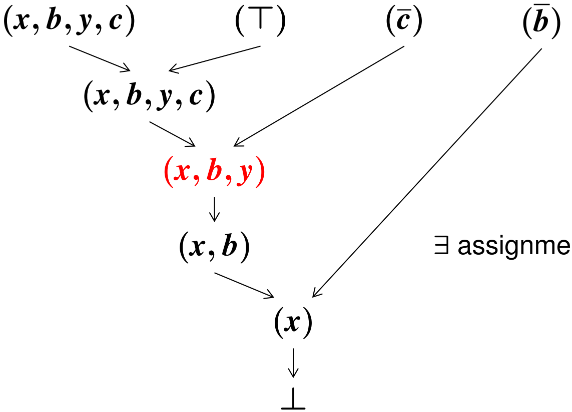
(x)

$\bot$

Strategy Extraction from LDQ-Refutations

$\exists a \ \forall x \ \exists b \ \forall y \ \exists c$

Strategy Extraction



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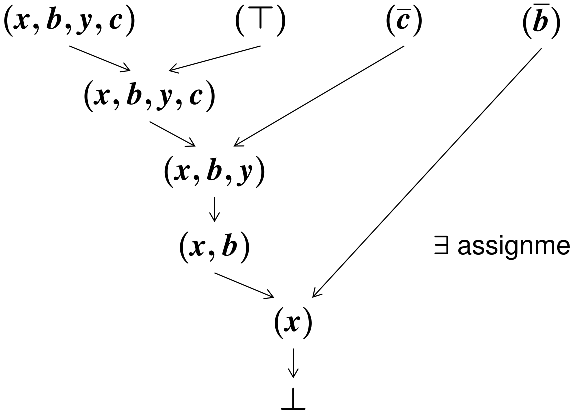
Strategy Extraction from LDQ-Refutations

Strategy Extraction

$\exists a\ \forall x\ \exists b\ \forall y\ \exists c$



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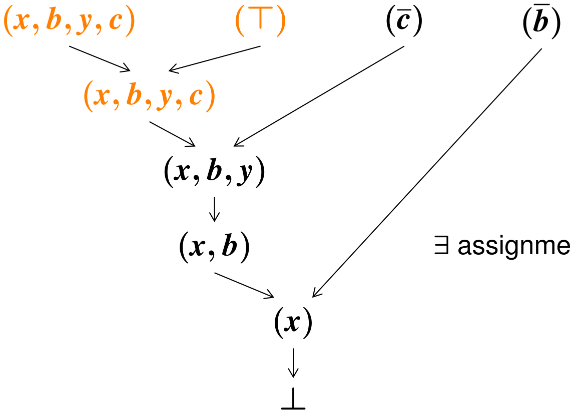

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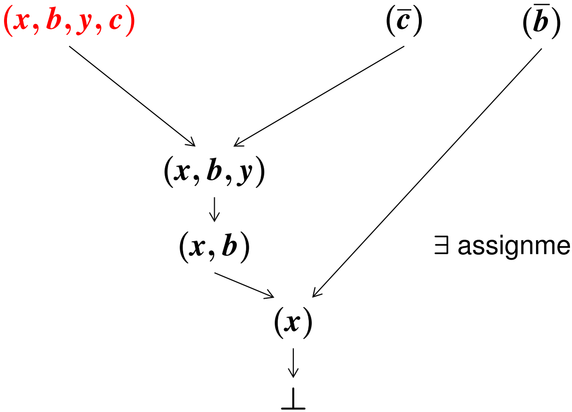
Strategy Extraction from LDQ-Refutations

$\forall x \; \exists b \; \forall y \; \exists c$

Strategy Extraction



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Strategy Extraction from LDQ-Refutations

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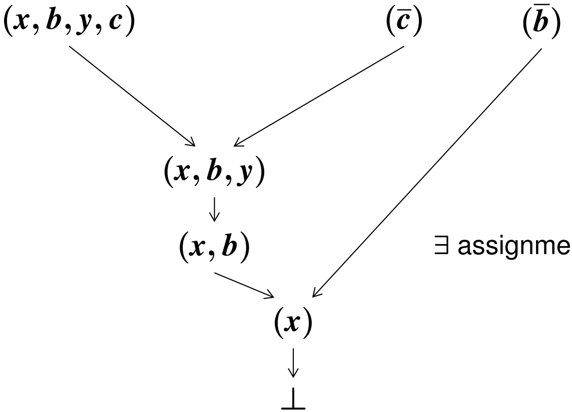

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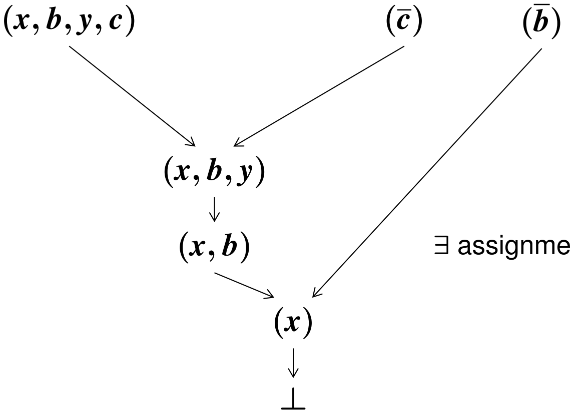

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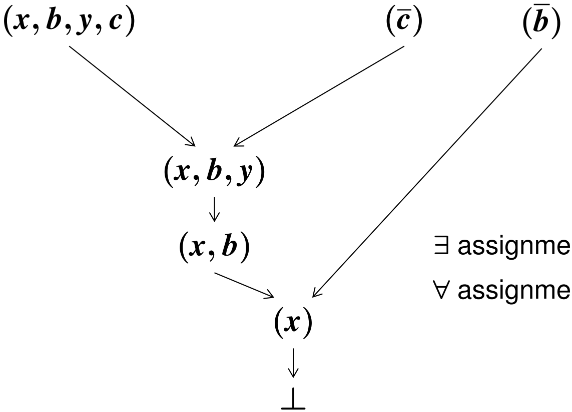

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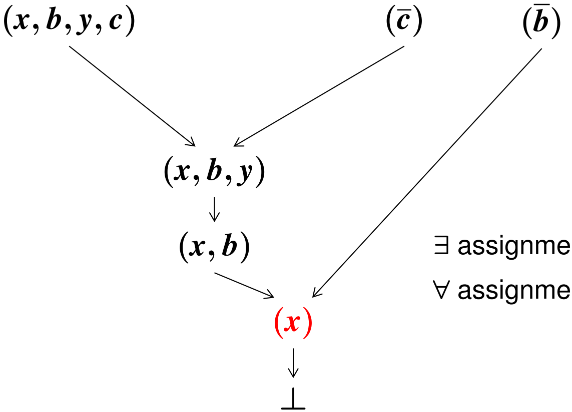

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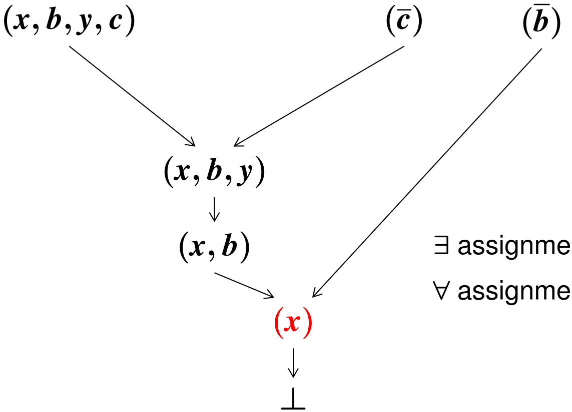

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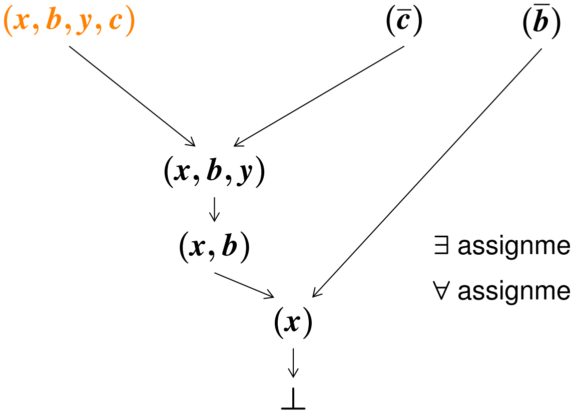

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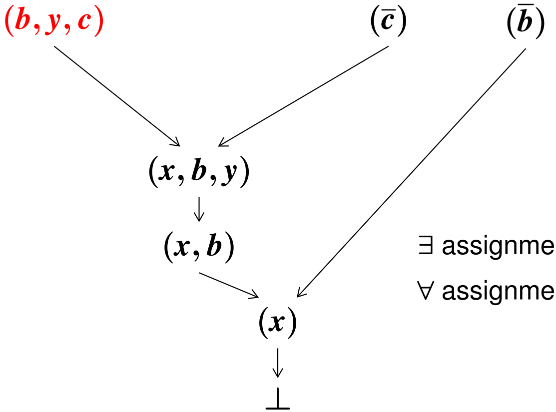

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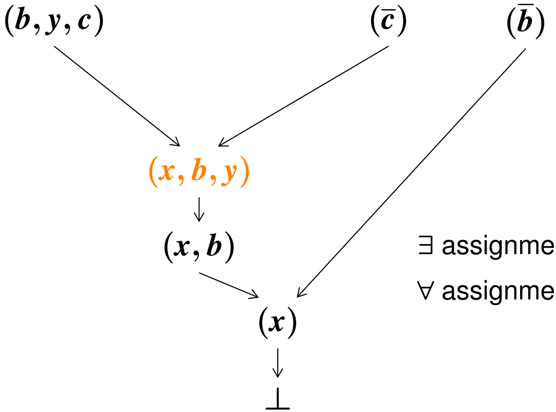

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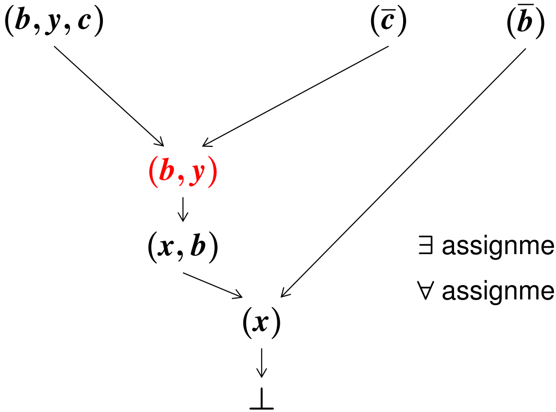

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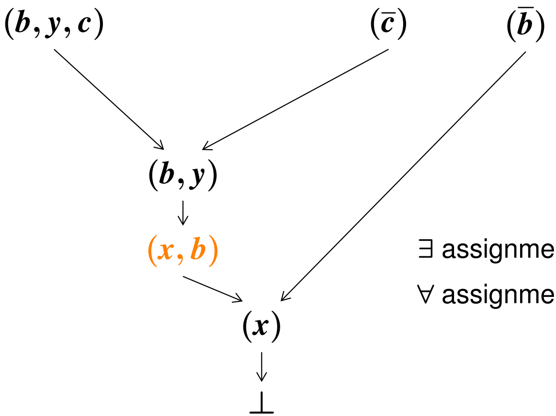

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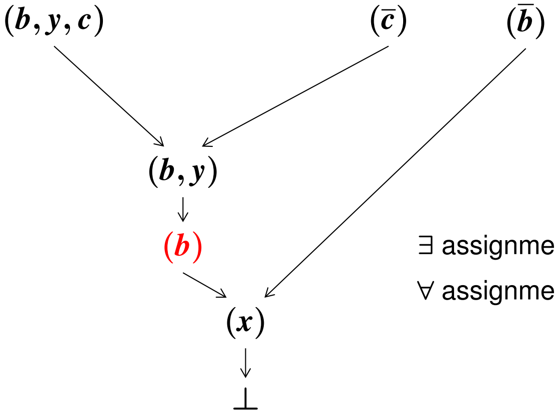

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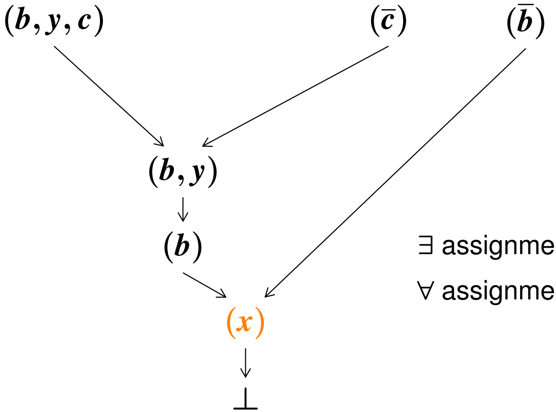

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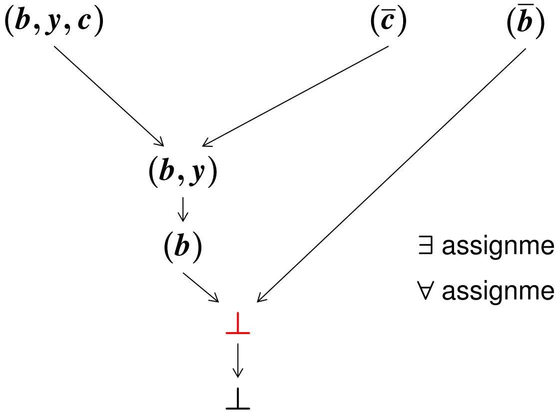

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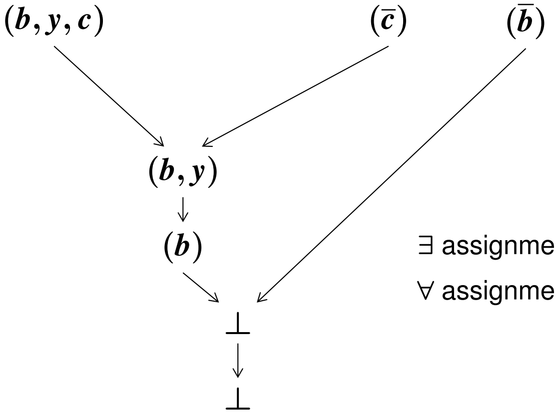

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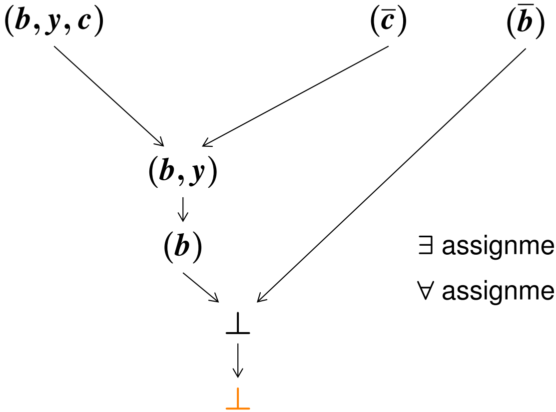

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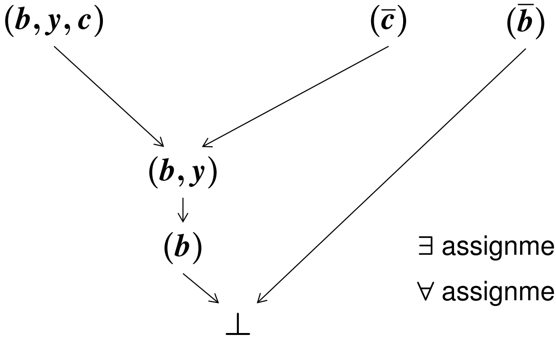

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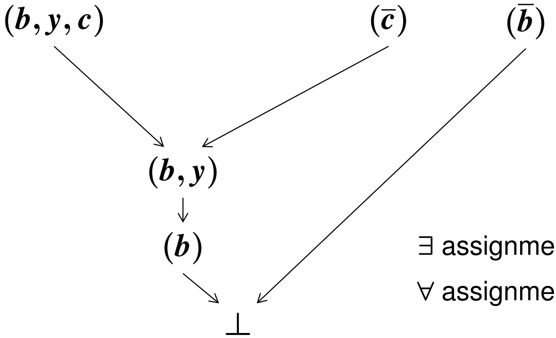

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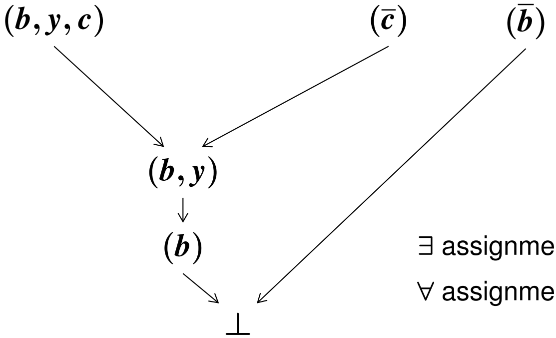

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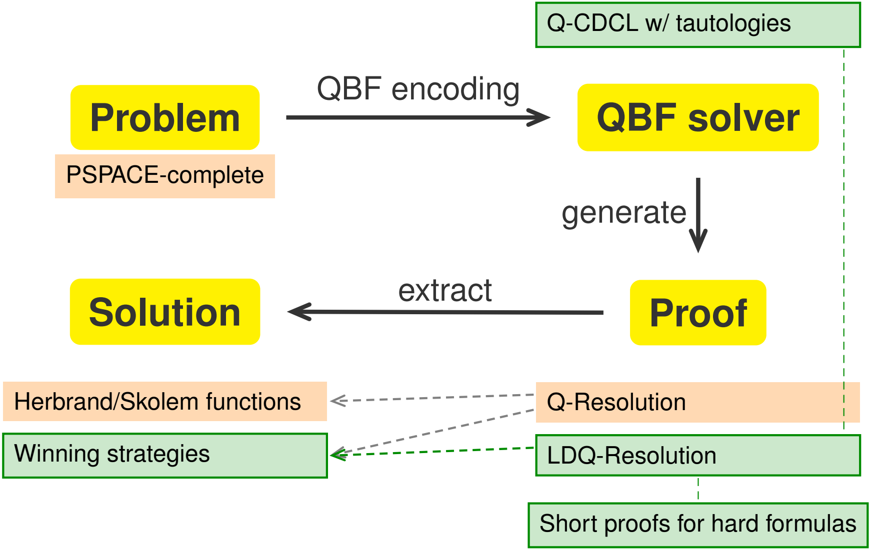
…and so on

Summary and Outlook



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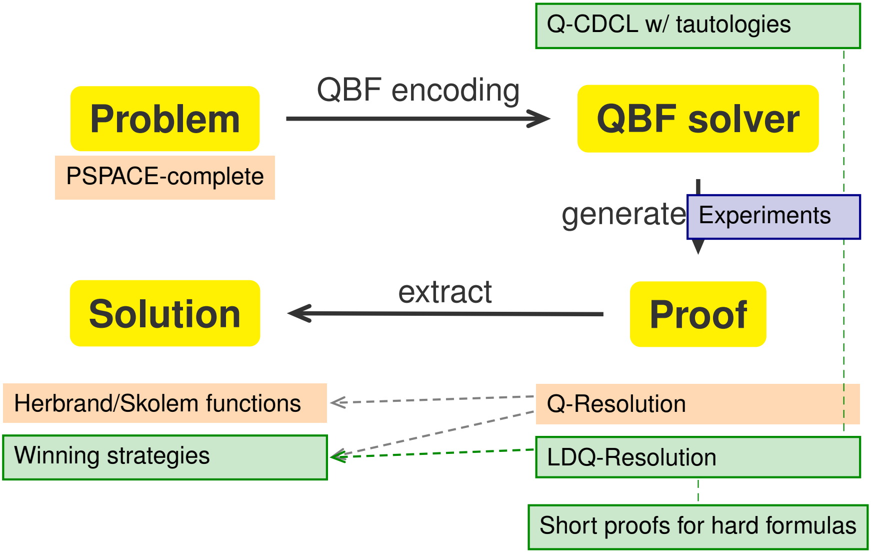

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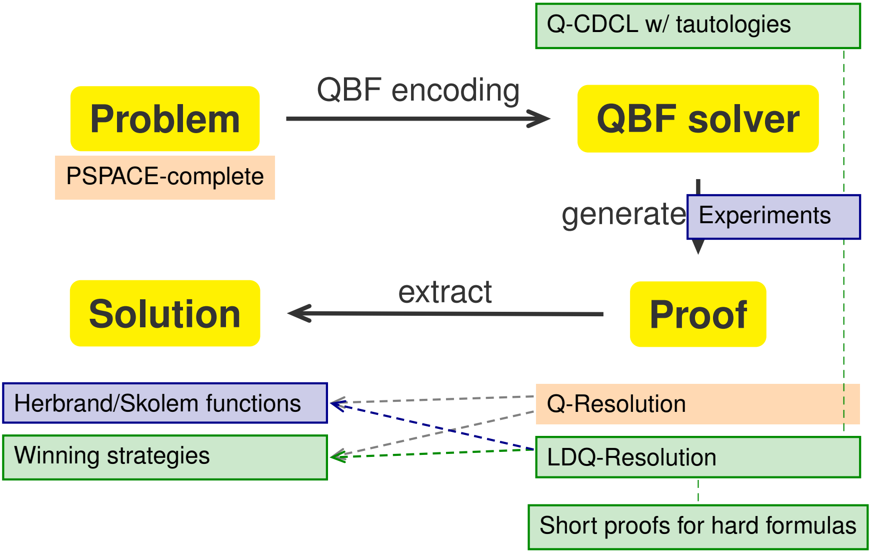

Short Proofs for Hard Formulas

Family $(\varphi_t)_{t\geq 1}$ of QBFs in PCNF with prefix

$$\exists d_0 d_1 e_1 \forall x_1 \exists d_2 e_2 \forall x_2 \exists d_3 e_3 \cdots \forall x_{t-1} \exists d_t e_t \forall x_t \exists f_1 \cdots f_t$$

and matrix:

K_0	$\overline{d_0}$	K_1	$d_0 \vee \overline{d_1} \vee \overline{e_1}$	
K_{2j}	$d_j \vee \overline{x_j} \vee \overline{d_{j+1}} \vee \overline{e_{j+1}}$	K_{2j+1}	$e_j \vee x_j \vee \overline{d_{j+1}} \vee \overline{e_{j+1}}$	$j = 1, \ldots, t-1$
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\end{array}$$

- Any Q-refutation for φ_t is exponential. [**?**]
- Q-resolution plus resolution over ∀ variables yields polynomial refutations. [**?**]
- Polynomial LDQ-refutation is possible. [This work]

Experimental Results

LDQ-resolution in the search-based QBF solver DepQBF:

- Preprocessed benchmarks from QBF Evaluation 2012.
- DepQBF with traditional Q-resolution solves more benchmarks:

QBFEVAL'12-pre (276 formulas)	
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115 solved by both:	DepQBF-LDQ	DepQBF
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- Future work: more detailed experimental analysis.